

Fundamentals of Fracture Mechanics

Lecture Notes Transcription

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1 Introduction to Fracture Mechanics

1.1 Basic Concepts

Fracture Mechanics depends on impact resistance. It helps us to analyze the equilibrium of resistance and impact (load).

Limit State Analysis:

- Defines the final loading condition which is safe.
- Used to evaluate whether a structure containing a pre-crack will break or at least suffer under the applied loading situation.
- Determines if crack initiation and propagation will happen.

1.2 Influencing Factors of Impact

To improve resistance, we must consider:

- Texture design and orientation.
- Grain size (Hall-Petch relation).
- Phase composition.
- Inner defects (e.g., in weld seams, oxidation).
- Temperature (Temperature dependency of fracture).

2 Theory of Fracture

2.1 Early Theories

Definition

Griffith (Energy Principle): Energy available for crack growth is sufficient to overcome the resistance of the material.

Definition

Irwin (Energy Release Rate): Rate of change in potential energy with the crack area for a linear elastic material. Depends on crack length.

2.2 Mechanics of Metals

The stress state is often described by tensors.

Stress Tensor ($\underline{\sigma}$):

$$\underline{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

For uniaxial tension (e.g., in z-direction), the tensor simplifies significantly.

Equivalent Stress (σ_e): Based on Von Mises yield criterion:

$$\sigma_e = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \quad (1)$$

3 Fracture Mechanisms

3.1 Ductile Fracture

Ductile fracture involves significant plastic deformation.

1. **Phase A (Void Initiation):** Pores initiate at hard particles (inclusions) in the microstructure or at interfaces (e.g., Ferrite-Martensite).
2. **Phase B (Void Growth):** Voids grow under hydrostatic stress.
3. **Phase C (Void Coalescence):**
 - Coagulation: Coalescence of two primary pores close to each other.
 - Shear bands: Due to strain localization connecting voids.

3.2 Cleavage Fracture (Brittle)

- Rapid propagation along specific crystallographic planes.
- Occurs mostly in BCC (Body Centered Cubic) lattices.
- Triggered by dislocation accumulation at grain boundaries (GB) or precipitates, causing high stress concentration.
- **Failure Criterion:** Locally, the cleavage fracture stress must be overcome, and micro-cracks must be present.

3.3 Atomic View on Fracture

Fracture at the atomic level is overcoming the binding energy E_b .

$$E_b = \int_{x_0}^{\infty} P dx \quad (2)$$

Where P is the binding force and x_0 is equilibrium spacing.

Approximating the force-displacement with a sine wave:

$$P = P_c \sin\left(\frac{\pi x}{\lambda}\right) \approx P_c \left(\frac{\pi x}{\lambda}\right) \quad (\text{for small } x)$$

Theoretical cohesive stress (σ_c):

$$\sigma_c \approx \frac{E}{\pi} \quad (3)$$

Note: This atomistic formula overestimates the real value because real materials have defects.

4 Linear Elastic Fracture Mechanics (LEFM)

4.1 Griffith Theory

Only applicable for purely linear materials (brittle).

$$\sigma_f = \sqrt{\frac{2E\gamma_s}{\pi a}} \quad (4)$$

Where γ_s is surface energy and a is crack half-length.

Crack Growth Condition:

$$\frac{dW_E}{dA} > \frac{dW_S}{dA}$$

(Change in elastic energy > Change in surface energy).

4.2 Stress Intensity Factor (K)

K characterizes the stress field near the crack tip ($r \rightarrow 0$ singularity).

Key Formula

$$K_I = \sigma \sqrt{\pi a} \cdot Y(a/W)$$

Where Y is the geometrical correction factor.

- **Limit State:** $K_I = K_{IC}$
- K_{IC} is the **Fracture Toughness** (Material Property).
- If $K_I > K_{IC}$, unstable crack growth occurs.

4.3 Energy Release Rate (G)

For linear elastic materials:

$$G = \frac{\pi \sigma^2 a}{E} \quad (\text{Plane Stress})$$

$$G = \frac{\pi \sigma^2 a}{E/(1 - \nu^2)} \quad (\text{Plane Strain})$$

Relation between G and K :

$$G = \frac{K_I^2}{E'}$$

Where $E' = E$ (Plane Stress) and $E' = \frac{E}{1 - \nu^2}$ (Plane Strain).

4.4 Mixed Mode Fracture

Cracks can be loaded in three modes:

- **Mode I:** Opening (Tensile).
- **Mode II:** In-plane Shear (Sliding).
- **Mode III:** Out-of-plane Shear (Tearing).

Total Energy Release Rate:

$$G_{total} = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu}$$

5 Elastic-Plastic Fracture Mechanics (EPFM)

LEFM is not valid when there is significant plasticity at the crack tip.

5.1 Crack Tip Plasticity

The plastic zone size r_p can be estimated (Irwin approximation):

$$r_p \approx \frac{1}{\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2$$

5.2 J-Integral

The J-integral is a path-independent contour integral used to characterize the elastic-plastic crack tip field.

$$J = \int_{\Gamma} \left(W dy - T_i \frac{\partial u_i}{\partial x} ds \right) \quad (5)$$

For linear elastic material, $J = G$.

Experimental determination of J from Load (P) vs Displacement (u) curves:

$$J = \frac{\eta A_{pl}}{B_N b_0}$$

Where A_{pl} is the area under the plastic part of the load-displacement curve.

5.3 CTOD (Crack Tip Opening Displacement)

Another parameter for EPFM. The displacement at the original crack tip. Relationship (Small Scale Yielding):

$$\delta = \frac{K_I^2}{m \sigma_{ys} E'}$$

6 Testing and Statistical Analysis

6.1 Types of Tests

- **Charpy Impact Test:** Fast, cheap, qualitative. Measures energy absorbed (A_v). Used for transition temperature curves (T_{27J}).
- **Fracture Mechanics Tests:** Determine K_{IC} , J_{IC} , CTOD. Uses specimens like CT (Compact Tension) or SENB (Single Edge Notched Bend).

6.2 Master Curve Concept

Describes the scatter of fracture toughness in the transition region (Ferritic steels). Based on the "Weakest Link" theory.

Weibull Distribution for Failure Probability (P_f):

$$P_f = 1 - \exp \left[- \left(\frac{K_{JC} - K_{min}}{K_0 - K_{min}} \right)^4 \right] \quad (6)$$

- K_{min} usually fixed at $20 \text{ MPa}\sqrt{\text{m}}$.

- K_0 is the scale parameter (63.2% failure probability).

Temperature Dependence (Master Curve Equation):

$$K_{JC,med}(T) = 30 + 70 \exp[0.019(T - T_0)]$$

Where T_0 is the reference temperature where median toughness is $100 \text{ MPa}\sqrt{\text{m}}$.

6.3 Failure Assessment Diagram (FAD)

A tool to assess safety considering both brittle fracture and plastic collapse.

- Y-axis: $K_R = K_{applied}/K_{mat}$ (Brittle fracture ratio)
- X-axis: $L_R = F_{applied}/F_{yield}$ (Plastic collapse ratio)

The component is safe if the point (L_R, K_R) lies inside the failure envelope.

7 Fatigue

7.1 Definitions

Material fatigue is gradual damage under fluctuating loads well below the yield strength.

- **Stress Ratio R:** $R = \sigma_{min}/\sigma_{max}$
- **Stress Amplitude:** $\sigma_a = (\sigma_{max} - \sigma_{min})/2$
- **Mean Stress:** $\sigma_m = (\sigma_{max} + \sigma_{min})/2$

7.2 S-N Curve (Wöhler Line)

Plots stress amplitude (σ_a) vs number of cycles to failure (N).

$$\sigma_a = \sigma'_f (2N)^b$$

Shows the endurance limit (infinite life strength) for steel.

7.3 Crack Propagation Laws

Paris-Erdogan Law: describes stable crack growth (Region II).

Key Formula

$$\frac{da}{dN} = C(\Delta K)^m$$

Where:

- da/dN is crack growth rate per cycle.
- $\Delta K = K_{max} - K_{min}$ is the stress intensity range.
- C, m are material constants.

Lifetime Estimation: Integration of Paris law from initial crack size a_i to critical size a_c :

$$N = \int_{a_i}^{a_c} \frac{da}{C(\Delta\sigma\sqrt{\pi a}Y)^m}$$

7.4 Fatigue Stages

1. **Crack Nucleation:** Persistent slip bands, extrusions/intrusions at surface.
2. **Micro-crack growth:** Shear mode (Stage I).
3. **Macro-crack growth:** Tensile mode (Stage II), striation formation.
4. **Failure:** Final instability.